MATHEMATICAL MODEL OF THE MARKET

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Abstract.

The paper presents a methodology of creating a mathematical model of the market as a vector (multiobjective) problem of mathematical programming or a vector optimization problem (VOP). The model is structured in such a manner that it takes into account, first, the balance of supply and demand, secondly, purposefulness of each market participant (a manufacturer and a consumer). The methods based on normalization of criteria and principles of guaranteed result are applied to solve the VOP. The methodology of modeling is shown through solving practical tasks of vector optimization - models of the one-product market with two manufacturers and consumers.

Key words: Marketing Model, Modeling, Market, Supply and demand, Management, Decisions making, Vector optimization,

Introduction

Marketing model construction and simulation of its behavior is one of the main problems of econometrics. This question has been of much attention in the world’s advanced countries since the middle of the nineteenth century. The majority of market models were made on the basis of competitive equilibrium establishment, about which Walras wrote in his work (1874). In the 1950s Arrow, Debreu (1954), McKenzie (1954), Gale (1967), Nikaido (1959) gave the mathematical justification of a Walras’s hypothesis in their works. Further there were research works on improvement of models and their generalization. Their descriptions are given fully enough in the monographs of Morishima (1964), Nikaido (1968), and of contemporary authors, such as Scherer and Ross (1990), Tirole (1992). The majority of these works examined the balance of the cumulative supply and demand (that is market equilibrium), Nikaido (1968).

The models found the balance between supply and demand, but they could not be considered marketing models, as, first, they did not take into account competition both between manufacturers, and between consumers; secondly, they did not include purposefulness of the market participant actions (manufacturers and consumers), which is the basis of competition. Game models were also of no effect. Marketing model should include not only contradiction between manufacturers and consumers, but also contradiction (competition) between each manufacturer; the same can be said about consumers. As a rule, what is good for one manufacturer (consumer) is bad for another and vice versa.

Thus, the marketing model should reflect not only balance between supply and demand but also purposefulness of each market participant and their interrelation. So the vector (multiobjective) problem of mathematical programming is that mathematical model which can reflect balance along with the purposefulness of each market participant. Some methods of solution are developed. They are based on normalization of criteria and a principle of guaranteed result, Mashunin, (1986, 2000).

Thus, it is important to solve the following problems: to develop a marketing mathematical model (simulator). At first there must be a one-product model, then a multiproduct one;

to use marketing models in economic development of region, nation.

The purpose of the present work is to create a model of the market as a vector problem of mathematical programming, which includes the balance between supply and demand and the purposefulness of each market participant. The work is designed for solution of practical tasks of market modeling. For this purpose the work gives the methods of solution of vector optimization problems, which underlie model of the one-product market with
two manufacturers and two consumers (Model 2*2) and their testing.

1. Model construction of the market

The model examines the market with one product. Q manufacturers and L consumers of the product are participate:

\[ q = 1, Q, l = 1, L \]

- is an index of the manufacturers (firms).

\[ l = 1, L \] - is an index and set of the consumers. Fig. 1.

Fig. 1. Market scheme and the relationship with the model

To construct a mathematical model of the market let’s create for each manufacturer and consumer: a vector variable, restrictions imposed on production operation and their purposes (criteria) of functioning.

The manufacturers (firms)

\[ X = \{ x_{ql}, q = 1, Q, l = 1, L \} \] – is a vector variable, determining volume of production made by \( q \in Q \) firm and sold to \( l \in L \) consumer for some final period of time \( t \in T \); (the time is not specified);

- \( c_q \) – is a cost per unit established by \( q \in Q \) firm in the market (for all consumers is the same);

- \( c_{ql} \) – is an amount of money received by \( q \in Q \) firm in the market from \( l \in L \) consumer;

\[ f_q(X) = \sum_{l=1}^{L} c_{ql} x_{ql} \] – is the value, which characterizes the amount of money received by \( q \in Q \) firm in the market from all consumers.

Resource (costs) characteristics:

- \( i = 1, M_q \) - is an index and set of all resources used by \( q \in Q \) firm for production;

- \( i = 1, M_{mat} \) - is an index and set of material resources used for production, \( M_{mat} \in M_q \);

- \( a_{iq}, i = 1, M_{mat} \) - costs of \( i \in M_{mat} \) resource per unit of output produced by \( q \in Q \) firm;

We assume that there is a linear, functional dependence of costs increase on output:

\[ g_i(X) = \sum_{i=1}^{I} a_{iq} x_{q} \in M_{mat} \]

where \( g_i(X) \) - costs of \( i \in M_{mat} \) resource for the whole volume of production.

\[ q=1, Q \] equivalent \( q=1, 2, \ldots, Q \).
The following calculations are carried out similarly:
labour expenses (cost of labour) per unit of production. \( i = 1, M_{lab} \) - an index and set of labour resources used for production, \( M_{lab} \in M_q \);
direct costs connected with production capacities per unit of any product, \( M_{cap} \in M_q \).

With the above-mentioned calculations, the cost price per product unit (variable costs) is computed as follows:

\[
a_q = \sum_{i=1}^{M_{lab}} c_i a_i + \sum_{i=1}^{M_{cap}} c_i a_i + \sum_{i=1}^{M_{cap}} c_i a_i, \quad q = \overline{1,Q},
\]

where \( c_i \) – is cost of material, labour and capacity resources respectively.

The same scheme is used to calculate overheads (constant) per product unit \( a_{qo} \). In the whole cost price of unit production is estimated as:

\[
a_q = a_{qv} + a_{qo}, \quad q = \overline{1,Q}, \quad (1.1)
\]

We assume that there is also a linear, functional dependence of costs increase in (1.1) on output:

\[
\sum_{q=1}^{L} a_{qXq} \leq b_q, \quad \forall q \in Q.
\]

The purpose of any manufacturer is to sell the maximal volume of goods at the highest price possible in order to get the highest profits possible\(^2\). It can be described as a problem of mathematical (linear) programming:

\[
\max f_q(X) = \sum_{q=1}^{L} p_q X_{ql}, \quad \forall q \in Q.
\]

with restrictions (1.2), \( \forall q \in Q \).

The consumers

\( X = \{ X_{ql}, q = \overline{1,Q}, l = \overline{1,L} \} \) – is a vector variable, determining volumes of the product bought by \( l \in L \) consumer from \( q \in Q \) manufacturer (firm), which is the same with a vector variable, determining actions of the manufacturer.

The actions of the of the consumer are restricted by minimal \( b_{ql}^{\text{min}}, l = \overline{1,L} \) and maximal \( b_{ql}^{\text{max}}, l = \overline{1,L} \) volume of financial assets which he can allocate to buy a product from different firms:

\[
b_{ql}^{\text{min}} \leq \sum_{q=1}^{Q} c_q X_{ql} \leq b_{ql}^{\text{max}}, \quad l = \overline{1,L}, \quad (1.4)
\]

where \( c_q \) – is cost per unit established by \( q \in Q \) firm in the market.

The main "to buy - to not buy "actions of the consumer when he purchases a product are:

the price \( c_q \) established \( q \in Q \) by the manufacturer. The consumer tries to choose the product with \( c_q, \forall q \in Q \) as small as possible;

\(^2\) Currently, there are several alternative models of firm behavior: a model of profit maximization, maximize sales, maximize growth, a model of managerial behavior; maximization model value added (Japanese model). Seo (2000).
characteristics of the product: quality, location of outlets, time of access, advertising and so on.

Let’s notice, that the quality rating of the product $\theta_q$, which is established by all the consumers for the $q$-th manufacturer, can be fixed in 100-point scale, i.e. $\forall \theta_q \leq 100$. At the same time the consumer wants to choose the goods of the highest quality.

The goal of any consumer is to buy the necessary volume of goods of the highest quality at the lowest price possible. It can be expressed as a problem of mathematical (linear) programming (PMLP):

$$\text{min } f(X) = \sum_{q=1}^{Q} c_q x_{ql},$$

with restrictions (1.4), $\forall l \in L$.

The problem of mathematical programming (1.5) is the model of behavior of any $l \in L$ consumer. In the model (1.5) consumer takes into account quality and other characteristics of the product implicitly in its price.

Model of one-product market

As a whole, let’s present a mathematical model of the one-product market as a vector problem of mathematical (in this case linear) programming considering both the purposes of all manufacturers (1.3) and consumers of a product (1.5), and the restrictions imposed on their opportunities (1.2) and (1.4):

$$\text{opt } F(X) = \{ \text{max } F_1(X)=\text{max } f_q(X) = \sum_{l=1}^{L} p_q x_{ql}, \ q=1, Q \},$$

$$\text{min } F_2(X)=\{ \text{min } f(X) = \sum_{q=1}^{Q} c_q x_{ql}, \ l=1, L \},$$

$$\sum_{q=1}^{Q} c_q x_{ql} \leq b_{i}^\max, \ l=1, L,$$

$$\sum_{l=1}^{L} a_q x_{ql} \leq b_q, \ q=1, Q,$$

$$x_{ql} \geq 0, \ q=1, Q, \ l=1, L,$$

where $F(X)$ is a vector criterion function. Thus, (1.6)-(1.10) is problem of vector optimization, where $K=Q \cup L$ is the set of criteria - manufacturers and consumers, respectively;

in (1.6) $F_1(X)$ - vector criterion “$Q$” producers, each component of which is the manufacturer, maximizing their profits $f_q(X), \ q=1, Q$;

in (1.7) $F_2(X)$ - vector criterion “$L$” consumers who minimize their costs, due to the cost for purchased products;

(1.8) – is budgetary constraints (financial) opportunities “$L$” consumers;

(1.9) – is restriction concerning production capacities of the “$Q$” manufacturers;

(1.10) - limitations associated with no negative volumes sold.

In the model (1.6)-(1.10), quality and other characteristics of the product, which the consumer takes into account, are determined by the priority of this or that criterion. The concept of criterion priority is given Mashunin, (1986, 2000).

The following chapter contains in brief the methods of solving a vector problem of linear programming (1.6)-(1.10) based on normalization of criteria and a principle of guaranteed result. For more detail see Mashunin, (1986, 2000).

2. Methods of the solution of the problem of vector optimization

In general view a vector problem of mathematical programming or vector optimisation problem (VOP) is considered as follows:

$$\text{Opt } F(X) = \{ \text{max } F_1(X) = \{ f_k(X), \ \ k = 1, K \},$$

where $K=Q \cup L$ is the set of criteria - manufacturers and consumers, respectively; $f_k(X)$ is the vector criterion of the $k$-th objective function; $K$ is the set of criteria of objectives; $F_1(X)$ is the vector criterion of the manufacturer; $F_2(X)$ is the vector criterion of the consumer; $x_{ql}$ is the volume of goods of the manufacturer, the consumer costs, due to the cost for purchased products; $b_{i}^\max, \ l=1, L$ are the budgetary constraints (financial) opportunities “$L$” consumers; $b_q$ are restrictions concerning production capacities of the “$Q$” manufacturers; $x_{ql} \geq 0, \ q=1, Q, \ l=1, L$ are limitations associated with no negative volumes sold.
\[ \min F_2(X) = \{ f_2(X), k = 1, K \} \],
\[ G(X) \leq B, X \geq 0, \]
where \( X = \{ x_j, j = 1, N \} \) is the vector of unknown quantities;

\[ F_1(X) = \{ f_k(X), k = 1, K \} \]
is the vector criterion function, or criterion vector, its every component is to be maximised;

\[ F_2(X) = \{ f_k(X), k = 1, K \} \]
is the same but every component of the function is to be minimised;
\[ K_1 \cup K_2 = K \] is the set of indices; \( G(X) \leq B, or \{ g_i(X) \leq b_i, i = 1, M \} \) the vector of constraints.

Let’s assume, that the problem (2.1)-(2.3) belongs to convex problems, and the set of allowable points \( S \) in restrictions (2.3) is not empty and presents a compact.
\[ S = \{ X \in R^N | X \geq 0, G(X) \leq B \} \neq \emptyset \]

Then there exists a solution vector, \( X^* \), if the problem (2.1)-(2.3) is solved against each criterion individually.
To solve the problem (2.1)-(2.3) the axioms of equality, equivalence and criterion priority have been worked out, which are based on normalisation of criteria and the principle of a guaranteed result. This allows to divide a Pareto optimal set \( S^o \) into a point subset where all the criteria are equivalent (the subset contains one point \( X^o \in S^o \)) and subsets \( S^o_q, q = 1, K \), where a criterion \( q \in K \) has priority over the rest:
\[ \bigcup_{q=1}^{K} S^o_q \bigcup X^o = S^o, S^o_q \subset S^o \subset S, q = 1, K. \]

Therefore, the solution of the problems is reduced to selecting a point from the corresponding subset. Thus, the decision problem (2.1)-(2.3) is brought to the solution of the problem:

\[ \lambda^o = \max \min_{X} p_k^q \lambda_k(X), \]
\[ G(X) \leq B, X \geq 0, \]
where
\[ \lambda_k(X) = \{ f_k(X) - f_k^o \} / (f_k \gamma_k^o), k = 1, K, X \in S, \]
\[ \lambda_k(X) \in [0, 1] \] is the relative estimate of a point \( X \in S \) against the \( k \)-th criterion; \( f_k^o, k = 1, K \) - the worst value of the \( k \)-th criterion if VOP (2.1)-(2.3) is solved against each criterion individually; \( f_k^o, k = 1, K \) - the optimal value of the \( k \)-th criterion.

\[ f_k^o = \min_{X \in S} f_k(X), \forall k \in K, f_k^o = \max_{X \in S} f_k(X), \forall k \in K; \]
\[ \forall k \in K \] is the relative estimation \( \lambda_k(X), k = 1, K \) lays within the limits of \( 0 \leq \lambda_k(X) \leq 1 \), \( k = 1, K \); \( p_k^q \) - is the priority of the \( q \)-th criterion over the \( k \)-th, \( p_k^q = \lambda_q(X)/\lambda_k(X), \forall q \in K, k = 1, K \), with \( p_k^q = 1, \forall q \in K, k = 1, K \) if VOP is solved with the equivalent criteria. The vector of priorities \( P^q = \{ p_k^q, k = 1, K \} \), \( \forall q \in K \) is calculated in an interactive procedure of taking solutions from a ratio:

\[ \lambda(X^q)/\lambda_q(X^q) \leq p_k^q \leq \lambda(X_q^q)/\lambda_q(X_q^q), \forall q \in K, k = 1, K. \]

Entering of an additional variable \( \lambda = \max_{k \in K} p_k^q \lambda_k(X), (\lambda \leq p_k^q \lambda_k(X), \forall q \in K, k = 1, K) \) will transform a maximin problem (2.4) into one criterion \( \lambda \)-problem:

\[ \lambda^o = \max \lambda. \]
The $\lambda$-problem (2.8) is solved by standard methods.

Solution of the $\lambda$-problem (2.8) will result in getting a point of an optimum $X^o$ and maximal relative estimation $\lambda^o$, such as

$$\lambda^o \leq \lambda_d(X^o), \ k=1, K, \ X^o \in S, \ (2.9)$$

in VOP with equivalent criteria, where $p^q_k = 1, \ \forall q \in K, \ k=1, K$;

$$\lambda^o \leq p^q_k \lambda_d(X^o), \ \forall q \in K, \ k=1, K, \ X^o \in S, \ (2.10)$$

in VOP with the given priority of the $q$-th criterion over the rest.

Thus, $\lambda^o$ is the maximal lower level for all the relative estimations $\lambda_d(X^o), \ k=1, K$ (guaranteed result) in relative units, and point $X^o \in S$ is optimum against Pareto according to the theorem 2.3 (Mashunin, (1986, p.23)). Changing $p^q_k, \ \forall q \in K, \ k=1, K$, in (2.7) makes it possible to calculate any point from Pareto set with the given accuracy, Mashunin, (2013).

The solution of the vector optimization problem (2.1)-(2.3) with equivalent criteria and given priority of the $q$-th criterion over others according to Mashunin, (1986, 2013) for model of the one-product market is shown in Chapter 4.

3. Solution of the vector optimization problem, underlying the model of the one-product market

Let's examine the solution of VOP (1.6)-(1.10), representing the mathematical model of the one-product market. Solving the problem (1.6)-(1.10) with equivalent criteria on the basis of normalization of criteria and principle of guaranteed result, we shall calculate:

- a point of an optimum $X^o = [x_{ql}, \ q=\overline{1, Q}, \ l=\overline{1, L}]$, which determines output of a product sold by each manufacturer to each consumer;
- values of criterion functions $f_k(X^o), \ k=\overline{1, K}, \ K=Q \cup L$, including, $f_q(X^o), \ q=\overline{1, Q}$, which determine the income (benefits) of each manufacturer; $f_q(X^o), \ q=\overline{1, Q}$, which determine expenditures of each buyer;
- The maximal relative estimation $\lambda^o$, where

$$\lambda^o \leq \lambda_d(X^o), \ k=\overline{1, K}, \ X^o \in S,$$

where $\lambda_d(X^o) = (f_k(X^o)-f^o_k)/(f^o_k-f^o_k), \ k=\overline{1, K}, \ X^o \in S$ – is normalized criterion (the relative estimate), in which $f^o_k$ - is the best solution against criterion $k \in K, f^o_k$ - is the worst one accordingly, $K=Q \cup L$ – is the set of criteria.

The maximal relative estimation $\lambda^o$ can be interpreted as a maximum level of the mutual interests of all manufacturers and consumers in relative units. Any increase of interests (criterion) of any manufacturer or consumer changes for the worse for all the rest participants of the market (manufacturers and consumers). The received point is an optimum against Pareto.

- A vector function $F_1(X)=[f_q(X^o), \ q=\overline{1, Q}]$ is a function of supply (offer), and a vector function $F_2(X)=[f_l(X^o), \ l=\overline{1, L}]$ is a function of demand.

The theorem concerning the analysis of results of the VOP (2.1)-(2.3) with equivalent criteria (and vector function $F_1(X)$ and $F_2(X)$ analysis in particular) is given below.

Theorem 1. (Theorem of the most Contradictory criteria in VOP with equivalent criteria).

In a convex vector problem of mathematical programming (2.1)-(2.3) with equivalent criteria solved on the basis of normalization of criteria and a principle of guaranteed result, in an optimum point $X^o$ there will always be
two criteria with indexes of \( q \in K, \ p \in K \) (let’s call them the most contradictory criteria of all criteria \( k=\overline{1, K} \) ), for which the following equality is true:
\[
\lambda'' = \lambda_q(X^o), \ q, \ p \in K, \ X \in S, \quad (3.1)
\]
for the rest of criteria, the following inequalities are true:
\[
\lambda'' \leq \lambda_q(X^o), \ \forall k \in K, \ q \neq p \neq k. \quad (3.2)
\]
Let’s look into the methodology of the solution of the vector problem \( (4.6)-(4.10) \), simulating the one-product market in the form of test examples.

In the first test example the market is presented by two manufacturers and two consumers (we shall call the model of such market - model 2*2) with same parameters (with the same prices for the goods and costs). Model 2*2 was solved, first, with equivalent criteria, secondly, with the given priority of criterion, where the priority of criterion simulates affinity (distance) one of the manufacturers to the consumers or quality of the goods.

4. Model of the one-product market with two manufacturers and two consumers (Model 2*2)

4.1. Construction of the model of the one-product market 2*2

For the analysis of market structures consider the model \((1.6)-(1.10)\) for the most about my situation, when the market with the same item participate, two manufacturers, and two of the consumer of the product. The closest to this situation duopoly model Cournot, Stackelberg, Bertrand and Adiwarta, Seo (2000), which consider the relationship only two manufacturers (firms).

Let us enter designations:

\[a_1, \ a_2 \] – are volumes of a product sold by the first manufacturer, \(x_3, \ x_4 \) – the same for the second manufacturer to the first and the second consumer accordingly;

\[c_1 = c_2 = c_3 = c_4 = 10 \] – is the price for a product;

\[a_1 = a_2 = a_3 = a_4 = 8 \] – are the costs of production for both manufacturers;

\[p_1 = p_2 = p_3 = p_4 = (c_1-\alpha_1) = 2 \] - profit from production and sales of a product for both manufacturers;

\[b_{1i}^{min} = 700, \ b_{1i}^{max} = 1000, \ i = 1, 2 \] – are minimal and maximal volumes of financial assets, which the first and second consumer can allocate to buy the product from different firms.

\[b_{i} = 1000 \] – are financial opportunities of a firm for manufacturing the product, \(q = 1, 2\).

The entered designations indicate, that each firm can make no more than \(x_{q}^{max} = b_{q}/\alpha_{q} = 125, \ q = 1, 2 \) units of the product, hence:

considering the cost \(c_q x_q^{max} = 1250\) it is more, than minimal sum of financial assets, which one consumer can allocate to buy the product from different firms \(b_{1}^{min} + b_{2}^{min} = 700\), but it is less, than the first and second consumer allocate together \(b_{1}^{min} + b_{2}^{min} = 1400\); considering the sum as \(x_{1}^{max} + x_{2}^{max} = 250\) and its costs as \(c_q (x_{1}^{max} + x_{2}^{max}) = 2500\) it is more, than maximal sum of financial assets, which the first and second consumer can allocate to buy the product from different firms \(b_{1}^{max} + b_{2}^{max} = 2000\).

Let us present the mathematical model of the market of two manufacturers and two consumers (model 2*2) as a vector problem of linear programming:

\[
\text{opt} \ F(X) = \{ \max F_1(X) = \{ \max f_1(x) = p_1x_1 + p_2x_3, \ \max f_2(X) = p_2x_2 + p_4x_4 \},
\]
\[
\min F_2(X) = \{ \min f_2(X) = c_3x_3 + c_4x_4 \}, \quad (4.1)
\]

with restrictions
\[
700 \leq c_1x_1 + c_2x_2 \leq 1000, \quad 700 \leq c_3x_3 + c_4x_4 \leq 1000, \quad (4.3)
\]
\[
a_1x_1 + a_2x_2 \leq 1000, \quad a_1x_1 + a_2x_2 \leq 1000, \quad (4.4)
\]
\[
x_1, x_2, x_3, x_4 \geq 0. \quad (4.5)
\]

Let us solve the vector problem \((4.1)-(4.5)\) on the basis of normalization of criteria and principle of guaranteed result with equivalent criteria and given priority of criterion.
In the market model (4.1)-(4.5) depending on the changes of parameters of model - commodity prices and resource costs are considered:

- model with the same parameters that corresponds to the model of perfect competition;
- model of oligopoly in which the parameters can be changed, i.e. in some companies there is the possibility of domination over other firms at prices of resources and other parameters.
- model monopoly, which has one criterion (4.1) manufacturers and multiple criteria (4.2) users (in our example, two);
- model of monophony in which there are many criteria (4.1) manufacture lei and one criterion (4.2) of the consumer.

Below is a model of perfect competition and model of oligopoly in which the decision is submitted at the same criteria and specified the priority criteria.

### 4.2. Solution of a vector problem simulating the market with equivalent criteria

The economic essence of the solution of a vector problem (4.1) - (4.5) is that all the purposes (criteria) of both manufacturers and consumers are equalized as relative estimations in joint optimization.

Let us present the solution of the vector task (4.1)- (4.5) with equivalent criteria on the basis of normalization of criteria and principle of guaranteed result as a sequence of steps according to Mashunin (1986, 2013).

**Step 1, 2.** The problem (4.1)-(4.5) is solved against each criterion separately. Let us look for the best solution \((X_k^*)\) and the worst solution \((X_k^\circ)\), i.e. for \(\forall k \in K = Q\) is determined the maximum and minimum \((f_k^* = \max_{X \in S} f_k(X), f_k^\circ = \min_{X \in S} f_k(X), \forall k \in K_2)\); are searched accordingly.

The solution of the problem (4.1)-(4.5) results in:

**Criterion 1.**

max: \(X_1^* = \{x_1 = 62.5, x_2 = 62.5, x_3 = 37.5, x_4 = 37.5\}\),

\[ f_1(X_1^*) = 250.0, \quad f_2(X_1^*) = 150.0, \quad f_3(X_1^*) = 1000.0, \quad f_4(X_1^*) = 1000.0, \]

min: \(X_1^\circ = \{x_1 = 7.5, x_2 = 7.5, x_3 = 62.5, x_4 = 62.5\}\),

\[ f_1(X_1^\circ) = 30.0, \quad f_2(X_1^\circ) = 250.0, \quad f_3(X_1^\circ) = 700.0, \quad f_4(X_1^\circ) = 700.0. \]

**Criterion 2.**

max: \(X_2^* = \{x_1 = 37.5, x_2 = 37.5, x_3 = 62.5, x_4 = 62.5\}\),

\[ f_1(X_2^*) = 150.0, \quad f_2(X_2^*) = 250.0, \quad f_3(X_2^*) = 1000.0, \quad f_4(X_2^*) = 1000.0 \]

min: \(X_2^\circ = \{x_1 = 62.5, x_2 = 62.5, x_3 = 7.5, x_4 = 7.5\}\),

\[ f_1(X_2^\circ) = 250.0, \quad f_2(X_2^\circ) = 30.0, \quad f_3(X_2^\circ) = 700.0, \quad f_4(X_2^\circ) = 700.0. \]

**Criterion 3.**

min: \(X_3^* = \{x_1 = 35.0, x_2 = 50.0, x_3 = 35.0, x_4 = 50.0\}\),

\[ f_1(X_3^*) = 170.0, \quad f_2(X_3^*) = 170.0, \quad f_3(X_3^*) = 700.0, \quad f_4(X_3^*) = 1000.0. \]

max: \(X_3^\circ = \{x_1 = 50.0, x_2 = 50.0, x_3 = 50.0, x_4 = 50.0\}\),

\[ f_1(X_3^\circ) = 200.0, \quad f_2(X_3^\circ) = 200.0, \quad f_3(X_3^\circ) = 1000.0, \quad f_4(X_3^\circ) = 1000.0. \]

**Criterion 4.**

min: \(X_4^* = \{x_1 = 50.0, x_2 = 35.0, x_3 = 50.0, x_4 = 35.0\}\),

\[ f_1(X_4^*) = 170.0, \quad f_2(X_4^*) = 170.0, \quad f_3(X_4^*) = 1000.0, \quad f_4(X_4^*) = 700.0. \]

max: \(X_4^\circ = \{x_1 = 50.0, x_2 = 50.0, x_3 = 50.0, x_4 = 50.0\}\),

\[ f_1(X_4^\circ) = 200.0, \quad f_2(X_4^\circ) = 200.0, \quad f_3(X_4^\circ) = 1000.0, \quad f_4(X_4^\circ) = 1000.0. \]

The economic sense of the solution of the vector problem (4.1)-(4.5) at the first step is that every market participant are given the most favorable conditions, i.e. calculations take into account only their restrictions and
determine their optimum opportunities. In the result we receive the optimum solutions \( f_k^* = f_k(X_k^*) \), \( k = 1, K \) - which are purposes every market participant wants to achieve.

Step 3. The standard normalization of criteria is carried out:
\[
\lambda_d(X) = \frac{(f_k(X) - f_k^*)}{(f_k^* - f_k^o)}, \quad k = 1, K, \quad X \in S.
\]

In the problem (4.1)-(4.5) \( \forall k \in K \) the relative estimation \( \lambda_d(X), k = 1, K \) lies within the limits of \( 0 \leq \lambda_d(X) \leq 1, \forall k \in K \). At this step the analysis of optimum points is carried out. (Note, the points lye on the frontier of Pareto set).

Let us carry out the analysis of the results of the solution against each criterion on the basis of normalization:
\[
\lambda_d(X^* ) = \frac{(f_k(X^* ) - f_k^*)}{(f_k^* - f_k^o)}, \quad (q, k) = 1, 2, 3, 4.
\]

The economic essence of the solution of the vector problem (4.1)-(4.5) at the third step is that the purposes of each market participant are leveled in numerical value. In the result we receive the normalized purposes \( \lambda_k^* = \lambda_d(X_k^*) \) = 1 (100%), \( \forall k \in K \), which each market participant wants to achieve.

Step 4. Construction of a \( \lambda \)-problem. Let us \( \forall X \in S \) determine a level \( \lambda = \min_{k \in K} \lambda_d(X) \) or \( \lambda = \lambda_d(X), k = 1, K \) and maximize it against \( X \in S \). The result is a maximin problem with normalized criteria,
\[
\lambda^* = \max_{\lambda \in \mathbb{S}} \min_{k \in K} \lambda_d(X).
\]

This problem is transformed to a \( \lambda \)-problem:
\[
\lambda^* = \max \lambda,
\]
\[
\lambda - (f_k(X) - f_k^*)/(f_k^* - f_k^o) \leq 0, \quad k = 1, K,
\]
\[
G(X) \leq B, \quad X \geq 0.
\]

In the appendix to the problem (4.1)-(4.5) \( \lambda \)-problem (4.6)-(4.8) will take the following form:
\[
\lambda^* = \max \lambda,
\]
with restrictions
\[
\lambda - (p_1x_1 + p_2x_2 - f_1(X_1^*))/(f_1(X_1^*) - f_1(X_1^o)) \leq 0,
\]
\[
\lambda - (p_3x_1 + p_4x_2 - f_2(X_2^*))/(f_2(X_2^*) - f_2(X_2^o)) \leq 0,
\]
\[
\lambda - (c_1x_1 + c_2x_2 - f_3(X_3^*))/(f_3(X_3^*) - f_3(X_3^o)) \leq 0,
\]
\[
\lambda - (c_3x_3 - f_4(X_4^*))/(f_4(X_4^*) - f_4(X_4^o)) \leq 0,
\]
\[
700 \leq c_1x_1 + c_2x_2 \leq 1000, \quad 700 \leq c_3x_3 + c_4x_4 \leq 1000,
\]
\[
a_1x_1 + a_2x_2 \leq 1000, \quad a_3x_3 + a_4x_4 \leq 1000,
\]
\[
x_1, x_2, x_3, x_4 \geq 0.
\]

The economic essence of the solution of the \( \lambda \)-problem (4.9)-(4.10) is that the \( \lambda \)-problem tends the criteria (relative estimations) of each market participant to the optimum \( \lambda(X_k^*) = 1, k = 1, K \).

Step 5. The solution of the \( \lambda \)-problem.

The \( \lambda \)-problem is a standard problem of convex programming and the standard algorithms of optimization are used to solve it.

The solution of the \( \lambda \)-problem results in receiving an optimum point \( X^* \), the values of objective function \( f_k(X^*) \), \( k = 1, K \) and maximal relative estimation \( \lambda^* \), which meets the conditions (2.9), i.e. \( \lambda^* \) is the maximal lower level for all relative estimations \( \lambda_d(X^*) \) - (guaranteed result in relative units).
The solution of the \( \lambda \)-problem (4.9)-(4.10). (Solution with equivalent criteria).

\[ \lambda^o = 0.6071, \quad X^o = \{ x^o_1 = 40.893, x^o_2 = 40.893, \quad x^o_3 = 40.893, x^o_4 = 40.893 \}. \]

Objective functions (criteria) in the optimum point \( f_k(X^o), \ k = 1,K \):

\[ f_1(X^o) = 163.572, \quad f_2(X^o) = 163.572, \quad f_3(X^o) = 817.858, \quad f_4(X^o) = 817.858, \]

Resource Costs \( A(X^o) \):

Financial costs 1, 2 consumers: \( c_1 x^o_1 + c_2 x^o_2 = 817.858, \quad c_3 x^o_3 + c_4 x^o_4 = 817.858; \)

Financial costs 1, 2 Manufacturer: \( a_1 x^o_1 + a_2 x^o_2 = 654.28, \quad a_3 x^o_3 + a_4 x^o_4 = 654.28. \)

Relative valuations at optimum \( \lambda_d(X^o), \ k = 1,K \):

\[ \lambda_1(X^o) = 0.60714, \quad \lambda_2(X^o) = 0.60714, \quad \lambda_3(X^o) = 0.60714, \quad \lambda_4(X^o) = 0.60714, \]

that meet the conditions of (2.9) \( \lambda^o \leq \lambda_d(X^o), q = 1,2,3,4 \), but all the criteria are contradictory according to (2.10). Any improvement (increase) of one of them will results in deterioration of the others.

4.3. Model of the market 2*2, with the given priority of criterion

The result received in the previous Chapter, is determined on the basis of criterion equivalence in the problem (4.1)-(4.5), i.e. criteria of both manufacturers and consumers do not a priority (preference) over each other. In real conditions there can be the priority of some criterion, for example, let us assume for the problem (4.1)-(4.5) that the second manufacturer \( q=2 \) is closer to the consumers than the first one, i.e. he has a priority in sales over him, \( q \in K \) (K=4 in the VOP (4.1)-(4.5)). In this case the solution is taken out of a subset of points with priority on the \( q \)-th criterion \( S^q \subseteq S^o \), lying between points \( X^o \) and \( X^q \), \( q \in K \) in Pareto set \( S^o \). The axiom of the choice of the criterion \( q \in K \) priority and of the method of solution of a vector problem with given priority of criterion based on this axiom are stated by Mashunin, (1986, 2013). Let us present this method as a sequence of steps on the example of the problem (4.1)-(4.5).

Step 1. The problem (4.1)-(4.5) with equivalent criteria is solved. The algorithm of the solution is given in the previous Chapter.

The solution result in receiving:

points of an optimum \( X^o_k, \ k = 1,K \) and value of criterion functions in these points \( f_k(X^o_k), \ k = 1,K \), which are the frontier of Pareto set;

the worst unchangeable part of criterion \( f_k, \ k = 1,K \);

The solution of the maximin problem and \( \lambda \)-problem, made on its basis, results in receiving \( X^o \) – which is the point of an optimum of the solution of VOP with equivalent criteria and maximal estimation \( \lambda^o \), which is the guaranteed level for all the criteria in relative units.

Let us calculate values of all the criteria \( q \in K \) in each point \( X^o_k, \ k = 1,K \):

\[ \{ f_k(X^o_k), \ q = 1,K \}, \ k = 1,K, \]

which show the value of each \( q \in K \) criteria on moving from one optimum point to another.

In the point \( X^o \) we shall calculate value of criteria \( f_k(X^o) \) and all relative \( \lambda_d(X^o), \ k = 1,K \), which meet the inequality \( \lambda^o \leq \lambda_d(X^o), \ k = 1,K, \) \( X^o \in S^o \), i.e. the point \( X^o \) is in the “center” of Pareto set in relative units, where all the criteria are above or equal to \( \lambda^o \). In other points \( X \in S \) the smallest in relative units criterion \( \lambda = \min_{k \in K} \lambda_d(X) \) is always less than \( \lambda^o \), i.e. \( \lambda^o \) is a maximum point of a dome made of the minimal relative estimations in set of points optimum on Pareto.

We shall calculate a matrix of relative estimations in points of an optimum out of (4.11)

\[ \lambda_q(X^o_k) = (f_q(X^o_k) - f^o_q) / (f^o_q - f^o_k), \ q = 1,K, \]

(4.12)
The matrix (4.12) shows values of all the criteria in relative units on the frontier of Pareto set. This information is very basic for further study of structure of Pareto set. The priority criterion \( q \in K \) is chosen on the basis of this information. The data of the \( \lambda \)-problem (4.6)-(4.8) are stored.

Step 2. As a priority, we choose the second criterion, i.e., the second producer has priority over other market participants. This was reported on the display. Computer processes the data that are related to the second criterion, and outputs to the display value criteria and relative valuations in the points \( X^*_2 \) and \( X^o \):

\[
\begin{align*}
&f_1(X^*_2) \leq f_2(X) \leq f_4(X^o), \quad \lambda_2(X^*_2) \leq \lambda_4(X) \leq \lambda_4(X^o); \quad (4.13) \\
&150.0 \leq f_1(X) \leq 163.572, \quad 0.54546 \leq \lambda_4(X) \leq 0.60714; \\
&250.0 \geq f_2(X) \geq 163.572, \quad 1.0 \geq \lambda_4(X) \geq 0.60714; \\
&1000.0 \geq f_3(X) \geq 817.858, \quad 0.0 \leq \lambda_4(X) \leq 0.60714; \\
&1000.0 \geq f_4(X) \geq 817.858, \quad 0.0 \leq \lambda_4(X) \leq 0.60714; \\
\end{align*}
\]

The vector of priorities of the second criterion over the others of \( P^2_k(X) = \lambda_2(X)/\lambda_4(X), k=1,2,3,4 \) in the points of extremum \( X^*_2 \), \( X^o \) is calculated, i.e. the limits of change of a vector of priorities are determined:

\[
P^2_k (X^*_2) \leq P^2_k \leq P^2_k (X^o), \quad k = 1,4.
\]

In the given problem the vector of priorities lies within the limits:

\[
P^2_k = \{1.8333 \geq p^2_1 \geq 1.0, \quad 1.0 = p^2_2 = 1.0, \quad \infty \geq p^2_3 \geq 1.0, \quad \infty \geq p^2_4 \geq 1.0\}.
\]

These data are the basic information for decision-making.

Step 3. After analyzing the priority criterion, the person, who makes the decision, specifies the numerical value \( f_\phi \) desirable to be received from the matrix (4.13): 250.0 \( \geq f_2(X) \geq 163.572 \). Let us assume that the desirable value of the second criterion is equal to \( f_2 = 200 \).

Step 4. The relative estimation is calculated as follows: \( \lambda_2 = (f_2 - f^o_2)/(f^*_2 - f^o_2) = 0.7727 \).

Step 5. Let us calculate the proportional factors \( \rho \) between \( \lambda_q(X^o) \), \( \lambda_{q'} \) \( \lambda_q(X^*_q) \) assuming the linear character of change of criterion \( f_q(X) \) in (4.11) and to a relative estimation \( \lambda_q(X) \) in (4.12) accordingly:

\[
\rho = (\lambda_q(X^o)) / (\lambda_q(X^*_q) - \lambda_q(X^o)), \quad q \in K.
\]

In our example it is: \( \rho = (\lambda_2 - \lambda_2(X^o_2))/(1 - \lambda_2(X^o_2)) = 0.4215 \).

Step 6. Assuming linear character of change of a vector of priorities \( P^q_k(X), k=1,K \), we calculate it for \( \lambda_2 \) using received factor of proportionality \( \rho \):

\[
p^q_k = p^q_k(X^o) + \rho(p^q_k(X^*_q) - p^q_k(X^o)), \quad k = 1, K, \quad q \in K.
\]

In our example:

\[
p^2_k = \{p^2_1 = 1.3512, p^2_2 = 1.0, p^2_3 = 2.0, p^2_4 = 2.0\}.
\]

Step 7. Construction of a \( \lambda \)-problem. The vector of priorities \( p^2_k, k=1,4 \) is entered into the \( \lambda \)-problem (4.6)-(4.8), which results in receiving a \( \lambda \)-problem with a priority of the \( q \)-th criterion

\[
\lambda^o = \max \lambda,
\]

\[
\lambda - p^q_k \lambda_q(X) \leq 0, \quad k = 1, K \quad \text{and} \quad G(X) \leq B, \quad X \geq 0
\]

Step 8. Solution of the \( \lambda \)-problem.

The solution results in:

Point of an optimum \( X^o = \{x^o_j, \quad j = 1, N\} \) and maximal relative estimation \( \lambda^o \) where \( \lambda^o \leq p^q_k(X^o) \lambda_q(X^o) \), \( k = 1, K \);
values of criterion functions $f_i(X^o)$, $k=1, K$ and relative estimations $\lambda_d(X^o)$, $k=1, K$: The value $f_q(X^o)$ is not usually equal to given $f_o$. The error of choice $f_q$ is defined by the error of linear approximation of the real change of relative estimations $\lambda_e(X^o)$, $k=1, K$ and the vector of priorities $p^q_k(X), k=1, K, q \in K$, carried out at steps 5 and 6.

If the error $\Delta f_q = |f_q(X^o) - f_o|$ is less than $\Delta f$, then one should move to step 10, if $\Delta f_q \geq \Delta f$, then the next step is carried out.

(To solve our example one step is made, the results are at the end of the algorithm).

Step 9. The vector of priorities of criterion $q \in K$ is determined in point $X^o$

$$p^q_k(X^o) = \lambda_q(X^o)\lambda_d(X^o), \; k=1, K, \; q \in K.$$  

The new vector of priorities is under construction

$$p^q_{kw} = (p^q_k + p^q_l(X^o))/2, \; k=1, K, \; q \in K.$$  

Move to step 7. Here the $\lambda$-problem is solved with a new vector of priorities $p^q_{kw}, \; k=1, K, \; q \in K$ instead of $p^q_k, \; k=1, K, \; q \in K$.

Step 10. Will the VOP be solved with priority criterion of another value? If the answer is "yes", then move to step 3, if "no ", the following step is carried out

Step 11. Will the VOP be solved with another priority criterion? If the answer is "yes", then move to a step 2, if “no” follow the next step.

Step 12. The end.

At step 8 the solution of the $\lambda$-problem results in the following:

$$\lambda^o = 0.7678, X^o = \{x^o_1 = 42.88, x^o_2 = 34.62, x^o_3 = 45.59, x^o_4 = 53.86\}.$$  

Objective functions (criteria) in the optimum point $f_i(X^o), k = 1, K$:

$$f_1(X^o) = 155.02, f_2(X^o) = 198.91, f_3(X^o) = 884.83, f_4(X^o) = 884.83,$$

Resource Costs $A(X^o)$:

Financial costs 1, 2 consumers: $c_1x^o_1 + c_2x^o_2 = 884.83, c_3x^o_3 + c_4x^o_4 = 884.83$;

Financial costs 1, 2 Manufacturer: $a_1x^o_1 + a_2x^o_2 = 620.05, a_3x^o_3 + a_4x^o_4 = 795.67$.

Relative valuations at optimum $p^q_k(X^o), k = 1, K$:

$$\lambda_1(X^o) = 0.56827, \lambda_2(X^o) = 0.76777, \lambda_3(X^o) = 0.3839, \lambda_4(X^o) = 0.3839.$$  

Check: $\lambda^o \leq p^2_1 \lambda_1(X^o), \lambda_2(X^o), p^2_3 \lambda_3(X^o), p^2_4 \lambda_4(X^o),$ where $\{p^2_1 \lambda_1(X^o) = 0.7678, \lambda_2(X^o) = 0.7678, p^2_3 \lambda_3(X^o) = 0.7678, p^2_4 \lambda_4(X^o) = 0.7678\}$.  

The error of linear approximation $\Delta f_q = |f_q(X^o) - f_o| = 198.91 - 200 = 1.09 \; (0.545 \%)$. Carrying out the subsequent steps of the algorithm can reduce it.  

(In the example the steps 10, 11 are not carried out)

The given example has shown a practical opportunity of choice of any point from Pareto set with the given accuracy.

5. Model of the one-product market with different parameters for the manufacturers

5.1. Construction of marketing model with different parameters (model of oligopoly) and its solution with equivalent criteria

Two manufacturers and two consumers of the product participate in the one-product market for a period $t \in T$. Let us consider the second case with the different in value parameters for two manufacturers (the resource expenses and cost) - model of oligopoly and without priority of criterion determining quality of a product. The designations and values of parameters are similar to 5.1. Only the differences are shown:
\[ c_1 = c_3 = 10, \quad c_2 = c_4 = 12 \quad \text{are prices for the product for the first and the second manufacturer accordingly;} \]
\[ a_1 = a_3 = 8, \quad a_2 = a_4 = 10 \quad \text{are expense for manufacture of a product for both manufacturers.} \]

These data are inserted in the VOP (4.1)-(4.5). The result shall be modeling model 2*2 with different parameters. Let us present algorithm for its solution with equivalent criteria.

Step 1, 2. The problem (4.1)-(4.5) is solved against each criterion separately. As a result of the solution we shall receive:

Criterion 1. \( \max: X^1 = \{x_1 = 62.5, x_2 = 62.5, x_3 = 31.0, x_4 = 31.0\} \).
\[ f_1(X^1) = 250.0, \quad f_2(X^1) = 124.0, \quad f_3(X^1) = 997.0, \quad f_4(X^1) = 997.0 \]
\[ \min: X^0 = \{x_1 = 10.0, x_2 = 10.0, x_3 = 50.0, x_4 = 50.0\} \].
\[ f_1(X^0) = 40.0, \quad f_2(X^0) = 200.0, \quad f_3(X^0) = 700.0, \quad f_4(X^0) = 700.0 \]

Criterion 2. \( \max: X^1 = \{x_1 = 32.1, x_2 = 32.1, x_3 = 50.0, x_4 = 50.0\} \).
\[ f_1(X^1) = 128.4, \quad f_2(X^1) = 200.0, \quad f_3(X^1) = 921.1, \quad f_4(X^1) = 921.1 \]
\[ \min: X^0 = \{x_1 = 62.5, x_2 = 62.5, x_3 = 6.25, x_4 = 6.25\} \].
\[ f_1(X^0) = 250.0, \quad f_2(X^0) = 25.0, \quad f_3(X^0) = 700.0, \quad f_4(X^0) = 700.0 \]

Criterion 3. \( \max: X^1 = \{x_1 = 35.55, x_2 = 45.62, x_3 = 30.17, x_4 = 37.31\} \).
\[ f_1(X^1) = 158.34, \quad f_2(X^1) = 135.37, \quad f_3(X^1) = 700.0, \quad f_4(X^1) = 903.97 \]
\[ \max: X^0 = \{x_1 = 48.2, x_2 = 48.3, x_3 = 43.17, x_4 = 35.39\} \].
\[ f_1(X^0) = 193.06, \quad f_2(X^0) = 157.12, \quad f_3(X^0) = 1000.0, \quad f_4(X^0) = 908.0 \]

Criterion 4. \( \min: X^1 = \{x_1 = 48.56, x_2 = 33.55, x_3 = 37.31, x_4 = 43.17\} \).
\[ f_1(X^1) = 158.34, \quad f_2(X^1) = 135.37, \quad f_3(X^1) = 904.0, \quad f_4(X^1) = 700.0 \]
\[ \max: X^0 = \{x_1 = 48.33, x_2 = 48.19, x_3 = 35.39, x_4 = 43.17\} \].
\[ f_1(X^0) = 193.06, \quad f_2(X^0) = 157.12, \quad f_3(X^0) = 908.0, \quad f_4(X^0) = 1000.0 \].

Step 3. The standard normalization of criteria and the analysis of the results of the solution against each criterion are carried out:

\[
\lambda(X^\ast) = \begin{pmatrix}
1.0 & 0.5658 & 0.0098 & 0.0098 \\
0.4212 & 1.0 & 0.2629 & 0.2629 \\
0.5636 & 0.6307 & 1.0 & 0.3201 \\
0.5636 & 0.6307 & 0.3201 & 1.0
\end{pmatrix}
\]

Step 4. Construction of a \( \lambda \)-problem in form of (4.9)-(4.10).

Step 5. The solution of the \( \lambda \)-problem (4.9)-(4.10) with equivalent criteria.

The solution of the \( \lambda \)-problem. (Solution with equivalent criteria).

\[
\lambda^\circ = 0.6111, \quad X^\circ = \{x_1^\circ = 42.0833, x_2^\circ = 42.0833, x_3^\circ = 32.9861, x_4^\circ = 32.9861\},
\]

Objective functions (criteria) in the optimum point \( f_i(X^\circ) \), \( k = 1, K \) :
\[ f_1(X^\circ) = 168.33, \quad f_2(X^\circ) = 131.94, \quad f_3(X^\circ) = 816.66, \quad f_4(X^\circ) = 816.66 \]

Resource Costs \( A(X^\circ) \):

Financial costs 1, 2 consumers: \( c_1 x_1^\circ + c_2 x_2^\circ = 816.66, \quad c_3 x_3^\circ + c_4 x_4^\circ = 816.66 \);

Financial costs 1, 2 Manufacturer: \( a_1 x_1^\circ + a_2 x_2^\circ = 673.33, \quad a_3 x_3^\circ + a_4 x_4^\circ = 659.72 \).

Relative valuations at optimum \( \lambda_3(X^\circ), k = 1, K \) :
\[ \lambda_1(X^\circ) = 0.6111, \quad \lambda_2(X^\circ) = 0.6111, \quad \lambda_3(X^\circ) = 0.6111, \quad \lambda_4(X^\circ) = 0.6111 \]

that is the conditions (2.9) \( \lambda^\circ \leq \lambda_i(X^\circ), k = 1, 2, 3, 4 \) are met, and according to (2.10) all the criteria are contradictory. Any improvement of one of them results in deterioration of the other.

Thus, the increase of the product price of the second manufacturer has resulted in decreasing the quantity of the sold products from 81.78 to \( x_2 + x_4 \) (see Unit 4.2) to 65.97 to \( x_2 + x_4 \). The difference of sales volume was compensated by the first manufacture whose price for the goods is less.
5.2. Marketing Model 2x2 with a priority of criterion, determining the quality of the goods

The models of the one-product market presented in the previous Chapters do not take into account the goods quality and solution is carried out on the assumption of equivalence of criteria in VOP (4.1)-(4.5). In real life the consumers pay no less attention to the goods quality than to its price. Therefore, in the basic model of the one-product market (2.6)-(2.10) the description of the goods quality appreciated by the consumers can be presented as a priority of criterion of the manufacturer with the higher goods quality. Let us examine how the goods quality influences results of modeling of the one-product market with two manufacturers and two consumers (4.1)-(4.5). Unit 5.3 presents the solution with a priority of the second criterion in model (4.1)-(4.5). This approach will be used for estimation of the goods quality. Let us carry out the analysis of the problem solution with equivalent criteria given in the previous chapter and compare it with the results of the solution of model 2x2 with identical parameters. The results show, that sales volumes of the second manufacturer (q=2) has decreased from x2+x3=81.786 (the value of Unit 5.2) to x2+x3=65.97 (see previous unit). As a consequence, the profits of the second manufacturer have fallen. There are the questions: "Is it possible to increase sales volume at the expense of quality?" and "How much should the quality of goods be improved to make level the profits of the first and second manufacturers?"

Let us present the solution of VOP (4.1)-(4.5) with a priority of the second criterion as a sequence of steps to give the answers to these questions.

Step 1. VOP (4.1)-(4.5) with equivalent criteria is solved. The algorithm is given in the previous unit.

The data of λ-problem (4.6)-(4.8) are stored.

Step 2. The second criterion is chosen as the priority one. The data connected to the second criterion is being processed:

\[ f_1(X^+_2) \leq f_1(X) \leq f_1(X^-), \quad \lambda_2(X^+_2) \leq \lambda_2(X) \leq \lambda_2(X^-) ; \]  
\[ 128.4 \leq f_1(X) \leq 168.34 , \quad 0.57 \leq \lambda_2(X) \leq 0.61112 ; \]
\[ 200.0 \geq f_2(X) \geq 131.94 , \quad 1.0 \geq \lambda_2(X) \geq 0.61112 ; \]
\[ 921.1 \geq f_2(X) \geq 816.66 , \quad 0.0 \leq \lambda_2(X) \leq 0.61112 ; \]
\[ 921.1 \geq f_2(X) \geq 816.66 , \quad 0.0 \leq \lambda_2(X) \leq 0.61112 ; \]

The vector of priorities of the second criterion over others \( p_k^2 (X) = \lambda_2(X)/\lambda_k(X), k=1,4 \) in extremum points \( X^+_2, X^- \) is determined:

\[ p_k^2 (X^+_2) = [ \lambda_2(X^+_2) ] / [ \lambda_k(X^+_2) ] = 1.7673 \]  
\[ p_k^2 (X^-) = [ \lambda_2(X^-) ] / [ \lambda_k(X^-) ] = 1.0 \]  
\[ p_k^2 (X^+_2) = [ \lambda_2(X^+_2) ] / [ \lambda_k(X^+_2) ] = 1.5855 \]  
\[ p_k^2 (X^+) = [ \lambda_2(X^+) ] / [ \lambda_k(X^+) ] = 1.5855 \]  
\[ p_k^2 (X^-) = [ \lambda_2(X^-) ] / [ \lambda_k(X^-) ] = 1 \]  
\[ p_k^2 (X^+) = [ \lambda_2(X^+) ] / [ \lambda_k(X^+) ] = 1 \]  
\[ p_k^2 (X^-) = [ \lambda_2(X^-) ] / [ \lambda_k(X^-) ] = 1 \]  
\[ p_k^2 (X^+) = [ \lambda_2(X^+) ] / [ \lambda_k(X^+) ] = 1 \]  
\[ i.e. \ the \ limits \ of \ change \ of \ a \ vector \ of \ priorities \ P_k^2 (X^+_2) \leq P_k^2 \leq P_k^2 (X^-), k=1,4. \]

In the present problem the vector of priorities lies within the limits:

\[ 1.7673 \geq P_1 \geq 1.0, \quad 1.0 = P_2 \geq 1.0, \quad 1.5855 \geq P_3 \geq 1.0, \quad 1.5855 \geq P_4 \geq 1.0. \]

These data are the basic information for taking the decision.

Step 3. After analyzing priority criterion, we determine numerical value \( f_4 \). Let us assume, that the desirable value of the second criterion is \( f_2 = 168.336. \)

Step 4. The relative estimation is calculated as follows: \( \lambda_2 = (f_2 - f_2^0)/(f_2^0 - f_2^0) \) = 0.6826.

Step 5. Let us calculate the factor of proportionality between \( \lambda_q(X) \), \( \lambda_q(X^+_2) \) assuming linear character of the change of criterion \( f_4(X) \) (4.1) and according to a relative estimation \( \lambda_q(X) \) (4.12): \( \rho = (\lambda_2 - \lambda_2(X^+_2))/(1 - \lambda_2(X^+_2)) = 0.5323. \)
Step 6. Assuming linear character of change of a vector of priorities \( p_k^q(X) \), \( k=1, \bar{K} \), we shall calculate it for 
\[ \lambda_k, k=1, \bar{K}, \] using factor of proportionality \( \rho \):
\[ P_k^q = P_k^q(X^o) + (P_k^q(X^o) - P_k^q(X^o)) \cdot \rho, k=1, \bar{K}, q \in \mathcal{K}. \]

In the given problem the vector of priorities determined as follows:
\[ P_2^2 = \{ p_1^2 = 1.7673, p_2^2 = 1.0, p_3^2 = 1.5855, p_4^2 = 1.5855 \}. \]

Step 7. Construction of a \( \lambda \)-problem. Let us enter the given vector of priorities \( p_k^2 \), \( k=1,4 \) into the data of \( \lambda \)-problem (4.6)-(4.8). The result is a \( \lambda \)-problem with a priority of \( q \)-th criterion:

Step 8. The solution of a \( \lambda \)-problem.
The results of a \( \lambda \)-problem are the following:
\[ \lambda^o = 0.8158, X^o = \{ x_1^o = 37.25, x_2^o = 31.22, x_3^o = 31.21, x_4^o = 44.45 \}, \]

Objective functions (criteria) in the optimum point \( f_k(X^o), k=1, \bar{K} :\)
\[ f_1(X^o) = 136.93, f_2(X^o) = 167.76, f_3(X^o) = 845.64, f_4(X^o) = 845.64, \]

Resource Costs \( A(X^o) \):
Financial costs 1, 2 consumers: \( c_1x_1^o + c_2x_2^o = 845.64 \), \( c_3x_3^o + c_4x_4^o = 845.64; \)
Financial costs 1, 2 Manufacturer: \( a_1x_1^o + a_2x_2^o = 547.74, \)
Relative valuations at optimum \( \lambda_k(X^o), k=1, \bar{K} :\)
\[ \lambda_1(X^o) = 4616, \lambda_2(X^o) = 0.8158, \lambda_3(X^o) = 0.5145, \lambda_4(X^o) = 0.5145. \]

Check: \( \lambda^o \preceq p_k^2 \lambda_1(X^o), \lambda_2(X^o), p_3^2 \lambda_3(X^o), p_4^2 \lambda_4(X^o), \)
where \( \{ p_1^2 \lambda_1(X^o) = 0.8158, p_2^2 \lambda_2(X^o) = 0.8158, p_3^2 \lambda_3(X^o) = 0.8158, p_4^2 \lambda_4(X^o) = 0.8158 \}. \)

Conclusions. The volumes of production of both manufacturers have matched and are equal to sales volume of purchases. The demand and supply are equal.

6. Analysis of the model of the one-product market with the aggregated criteria

Estimating model (2.6)-(2.10) arises a desire to join efforts of both the manufacturers in (2.6) and consumers (2.7), as such method is standard. Let us check the criteria on models 2*2 of the previous units before summing up them and solving a problem.

In Unit 4.1 the model of the one-product market with two manufacturers and two consumers is constructed as a vector problem. Its solution is given in Unit 4.2. Let us enter criteria with the sums on both manufacturers and consumers into VOP (4.1)-(4.5):
\[ \max f_3(X) = p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4, \] \( \min f_3(X) = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4. \)

Let us solve the problem (4.1)-(4.5) with criteria (6.1) and (6.2).
Criterion (6.1). \( \max X^o_s = \{ x_1 = 50.0, x_2 = 50.0, x_3 = 50.0, x_4 = 50.0 \}, \)
\[ f_1(X^o_s) = 200.0, \quad f_2(X^o_s) = 200.0, \quad f_3(X^o_s) = 1000.0, \quad f_4(X^o_s) = 1000.0, \]
\[ f_5(X^o_s) = 400.0, f_6(X^o_s) = 2000.0. \]

Criterion (6.2). \( \min X^o_s = \{ x_1 = 35.0, x_2 = 35.0, x_3 = 35.0, x_4 = 35.0 \}, \)
\[ f_1(X^o_s) = 140.0, \quad f_2(X^o_s) = 140.0, \quad f_3(X^o_s) = 700.0, \quad f_4(X^o_s) = 700.0, \]
\[ f_5(X^o_s) = 280.0, f_6(X^o_s) = 2000.0. \]
Criterion (7.2). **min**: $X^*_6 = \{x_1=35.0, x_2=35.0, x_3=35.0, x_4=35.0\}$.

$f_1(X^*_6) = 140.0$, $f_2(X^*_6) = 140.0$, $f_3(X^*_6) = 700.0$, $f_4(X^*_6) = 700.0$.

$f_5(X^*_6) = 280.0$, $f_6(X^*_6) = 2000.0$.

**max**: $X^*_6 = \{x_1=50.0, x_2=50.0, x_3=50.0, x_4=50.0\}$.

$f_1(X^*_6) = 200.0$, $f_2(X^*_6) = 200.0$, $f_3(X^*_6) = 1000.0$, $f_4(X^*_6) = 1000.0$.

$f_5(X^*_6) = 400.0$, $f_6(X^*_6) = 2000.0$.

We estimate the results on the relative estimated at optimum points $X^*_5, X^*_6, X^*_7, X^*_8$ respectively

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<td>-0.000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

The results in relative estimations show, that the limits of changes of production of the first and second manufacturers lie within the limits:

$200 \geq f_1(X) \geq 140$, $0.773 \geq \lambda_3(X) \geq 0.5$; $200 \geq f_2(X) \geq 140$, $0.773 \geq \lambda_2(X) \geq 0.5$.

Thus, the aggregated criterion (6.1), (6.2) does not reflect in full measure the purposes of the manufacturers, which lie within the limits: $1.0 \geq \lambda_1(X) \geq 0$. These drawbacks of the aggregated criterion were repeatedly mentioned in literature.

Let us analyze reasons for it. When solving VOP (4.1)-(4.5) in a point of an optimum against the first criterion the second criterion reaches the optimum value. In the worst point against the first criterion the second criterion meets only 0.54 of its optimum value. In the worst point against the first criterion the second criterion meets only 0.54 of its optimum value. Thus, the purposes of the manufacturers are directly opposite, what is good for one is bad for another, and on the contrary, what is bad for one is good for another. This is competition. The sum of criteria cannot reflect these inconsistent purposes.

Let us estimate the solution of the problem (4.1)-(4.5) with additional criteria (6.1), (6.2) and restrictions:

$1400 \leq c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \leq 2000$,

(a) $a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 \leq 2000$.

As a result of the decision of a $\lambda$-problem is received:

$\lambda^o = 0.5$, $X^o = \{x_1=42.5, x_2=42.5, x_3=42.5, x_4=42.5\}$,

$f_1(X^o)=170$, $f_2(X^o)=170$, $f_3(X^o)=850$, $f_4(X^o)=850$, $f_5(X^o)=340$, $f_6(X^o)=1700$.

$\lambda_1(X^o)=0.6364$, $\lambda_2(X^o)=0.6364$, $\lambda_3(X^o)=0.5$, $\lambda_4(X^o)=0.5$, $\lambda_5(X^o)=0.5$.

The received result also shows that the aggregated criterion inexactly reflects the purposes both manufacturers and consumers, whose general level $\lambda=\min_{k=K} \lambda_k(X), \forall X \in S$ is raised up to $\lambda^c=\min_{k=K} \lambda_k(X^c)=0.60714, X^c \in S$ in the results of the solution of the VOP (4.1)-(4.6), reflecting individual purposes.

**Conclusions.** The work presents the construction of model of the one-product market as a vector problem of mathematical programming. The model takes into account not only balance between supply and demand, but also purposefulness of each market participant. The methodology of modeling of the market is shown through the solution of a number of practical problems presented by vector problems of linear programming.
References

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